

Introduction

Collisional kinetic plasma models (multispecies Vlasov-Maxwell-Landau, etc.) provide high-fidelity simulation of plasma physics, but at great computational cost.

A great variety of model reductions to these systems exist:

- Chapman-Enskog based asymptotic expansions in Kn^{-1} (e.g. Braginskii [1])
- Ansatz-based extensions and regularizations of the Grad 13N Moment scheme [5]
- Dynamical low-rank methods evolve an ansatz for f as a low-rank matrix in x and v. [4]

These methods save computation compared to the full 6 dimensional kinetic equation. Making good use of reduced models requires knowing when they are appropriate to apply.

- **Hybridization**: Split up physical domain into pieces according to predetermined parameter cutoffs
- Adaptivity: Dynamically upscale and downscale plasma model according to local conditions

But first: analyze data from full kinetic simulations to discover where fluid-like models provide a good fit.

BGK Shocktube Problem

Neutral fluid BGK model in 1D1V provides a test case for analyzing data-driven moment closure techniques. Initial-boundary value problem:

 $\left(\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \left(\frac{\partial f}{\partial t}\right)_{\text{BGK}} = -\frac{f - f_M}{\tau},$ $x \in [0, L], t \in [0, \infty)$ $f(0, v, 0) = f(0, v, t) = \mathcal{M}(1, 0, p_L), \quad x \in [0, L/2)$ $f(L, v, 0) = f(L, v, t) = \mathcal{M}(1, 0, 1), \quad x \in [L/2, L]$

with Maxwellian boundary conditions $\mathcal{M}(n, nu, nT)$.



shocktube solution f and its χ metric

Figure: Schematic of the moment closure problem in 1D1V

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Sparse Identification of Nonlinear Moment Closure Dynamics

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Sparse Regression (SINDy) for BGK m_3

The SINDy technique [2] provides a method for discovering nonlinear dynamics from data. Minimization formulation: $\min_{\xi} \left(\|\mathbf{A}\xi - m_3\|^2 + \lambda \|\xi\|_1 \right)$ $\implies \tilde{m}_3 = \mathbf{A}\xi$

A large nonlinear predictor library A consists of products t_0 of lower moments and their spatial derivatives.

Dataset consists of 3 BGK full kinetic simulations with varying collision time τ and shock strength.



Vlasov-Maxwell Simulation of Kelvin-Helmholtz Instability

$$\partial_t f_\alpha + \mathbf{v} \cdot \nabla_\mathbf{x} f_\alpha + \frac{q_\alpha}{m_\alpha} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_\mathbf{v} f_\alpha = 0, \quad ($$

where α ranges over the particle species $\{e, i\}$, and the fields **E**, **B** are evolved with Maxwell's equations.

[3] and [6] simulate the KH instability. Here we use data from [3] on a hybridized domain:



(a) Schematic of the domain hybridization

(b) Maxwellian deviation χ

Figure: The KH instability evolution sees χ grow in the highlighted region, where the ion species is treated kinetically.

Braginskii-like Heat Flux Closures

A Chapman-Enskog expansion of the *collisional* version of (1)gives a perturbative closure relationship for the ion heat flux in terms of the temperature:

$$\mathbf{q}^{i} = \int f_{i} |\mathbf{v} - \mathbf{u}|^{2} \mathbf{v} \, d\mathbf{v}$$

$$\approx \sum_{\text{Braginskii}} -\kappa_{\parallel} \nabla_{\parallel} T_{i} - \kappa_{\perp} \nabla_{\perp} T_{i} + \kappa_{\wedge} (\hat{b} \times \nabla T_{i}) + O(\epsilon^{2}). \quad (2)$$

Note

A Chapman-Enskog expansion of (1) cannot be justified without a collision term; however the form of (2) suggests a function library to learn a moment closure for \mathbf{q}

Figure: The simple linear regression described above predicts the shape of the heat flux in both x and y

The time-integrated L^1 error is 43% in q_x and only 18% in q_y .



The magnetic field \mathbf{B} is almost exactly constant, so we fit heat flux closures of the form

$$q_x \approx -\kappa_\perp \partial_x T_i - \kappa_\wedge \partial_y T_i$$
$$q_y \approx -\kappa_\perp \partial_y T_i + \kappa_\wedge \partial_x T_i$$

using a least-squares approach,

$$\min_{\substack{\boldsymbol{\xi}_{\boldsymbol{x}} \\ \boldsymbol{\xi}_{\boldsymbol{y}}}} \| \left[n_i T_i \partial_x T_i \ n_i T_i \partial_y T_i \right] \cdot \boldsymbol{\xi}_{\boldsymbol{x}} - q_x \|^2,$$
$$\min_{\substack{\boldsymbol{\xi}_{\boldsymbol{y}} \\ \boldsymbol{\xi}_{\boldsymbol{y}}}} \| \left[n_i T_i \partial_y T_i \ n_i T_i \partial_x T_i \right] \cdot \boldsymbol{\xi}_{\boldsymbol{x}} - q_y \|^2.$$



Sparse Identification of Nonlinear Dynamics is likely not a suitable tool for moment closure problems, as evidenced by the failure of sparsity on even a simple moment closure problem. Asymptotic analysis such as the Chapman-Enskog expansion is a more promising route for the discovery of relationships between lower and higher moments. Even in a fully noncollisional plasma simulation, the Chapman-Enskog derived Braginskii closure accounts for 60-80% of the ion heat flux.

Open Questions:

- asymptotic expansion of Equation (1)?

- domains.

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Conclusion

Future Work and Open Questions

• Is there a different SINDy library of nonlinear functions that will give better results?

• Do the learned moment closures lead to a stable system? Can a SINDy type framework be designed which will ensure hyperbolicity of the resulting moment system? • Can the observed moderate quality of fit to a Braginskii-like moment closure be predicted by some other

Future Work:

• Repeat Kelvin-Helmholtz instability simulations with a range of collisionalities.

• Develop a generic moment closure evaluation tool to determine whether proposed moment closures can stably and accurately reproduce kinetic results.

References

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